MCA12

Reg. No.

First Semester M.C.A Degree Examination, January/February 2003

Master of Computer Applications
(New Scheme)
Discrete Mathematical Structues

Time: 3 hrs.] [Max.Marks : 100

Note: *i) Answer any FIVE full questions. (All questions carry equal marks.)*
ii) The symbols $\longrightarrow$, $\wedge$ $\neg$ and $\vee$ denote the logical connective implication, conjunction, negation and disjunction.

1. (a) Define the following
i) Truth table ii) Conjuntion iii) Disjunction iv) Proposition v) Tautology vi) Contradiction **(6 Marks)**

(b) Obtain the principal disjunctive normal form of
$P \longrightarrow ((P \longrightarrow Q) \wedge \neg(\neg Q \vee \neg P))$ **(7 Marks)**

(c) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$ and R are logically equivalent. **(7 Marks)**

2. (a) Define a power set. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(6 Marks)**

(b) A survey was conducted among 1000 people. Of these 595 are democrats, 595 wear glasses and 550 like ice-creams. 395 of them are democrats who wear glasses, 350 of them are democrats who like ice-creams and 400 of them wear glasses and like ice-creams, 250 of them are democrats who wear glasses and like ice - creams.

i) How many of them who are not democrats, do not wear glasses and do not like ice - creams ?

ii) How many of them are democrats who do not wear glasses and do not like ice - creams ? **(7 Marks)**

(c) In a debating club, 7 members A, B ... I desire to take part in a debate. In how many ways a list of speakers be arranged so that

i) A speaks immediately after B

ii) A speaks after B. **(7 Marks)**

3. (a) Define partially ordered set. Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(X \leq)$ **(7 Marks)**

(b) Let $S = \{1, 2, 3, 4, 5\}$ and $A = S \times S$. Define the following relation R on A : $(a, b)R(a', b')$ iff $ab' = a'b$. Then show that R is an equivalence relation. **(7 Marks)**

(c) Let $A = \{a, b, c, d, e\}$ and let R and S be the relations on A described by
$R = \{(a, a), (a, c), (a, e), (b, d), (c, a), (c, c), (c, d), (e, a), (e, c)\}$
$S = \{(a, b), (a, d), (b, a), (b, b), (b, d), (c, a), (c, b), (c, c), (d, b), (e, b), (e, d)\}$
Find the matrix of the transitive closure of RUV using Warshall's algorithm. **(6 Marks)**

4. (a) If R and S are equivalence relations on a set A, show that $R \cap S$ is an equivalence relation. **(6 Marks)**

(b) Find an explict formula for the sequence defined by $C_n = 3C_{n-1} - 2C_{n-2}$ with the initial conditions $C_1 = 5$ and $C_2 = 3$ **(6 Marks)**

(c) Let R be a relation from A to B and let A_1 and A_2 be subsets of A.

i) If $A_1 \leq A_2$ then $R(A_1) \leq R(A_2)$

ii) $R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$ **(8 Marks)**

5. (a) List all possible functions from $X = \{a, b, c\}$ to $Y = \{0, 1\}$ and indicate in each case whether the function is one-one, is onto and is one-one outo. **(8 Marks)**

(b) Let f, g, h be functions from N to N, where N is the set of natural numbers including zero such that set of natural numbers including zero such that

$$f(n) = n + 1, \quad g(n) = 2n$$

$$h(n) = \begin{cases} 0 & n \ is \ even \\ 1 & n \ is \ odd \end{cases}$$

Determine $fof, \ fog, \ gof, \ goh, \ hog, \ (fog) \ oh.$ **(6 Marks)**

(c) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 5 & 4 & 2 & 6 \end{pmatrix}$ be a permutation of A

i) Write p as a product of disjoint cycles.

ii) Compute p^{-1} and p^2

iii) Find the period of p, that is the smallest positive integer k such that $p^k = 1_A$ **(6 Marks)**

6. (a) Define an Euler and Hamiltonian graph with one example each. Give an example of a graph which is

i) Eulerian but not Hamiltonian

ii) Hamiltonian but not Eulerian

iii) Both Eulerian and Hamiltonian

iv) Neither Eulerian nor Hamiltonian **(6 Marks)**

(b) Show that graph G is a tree if and only if there is only one path between every pair of vertices in G. **(6 Marks)**

(c) Define a spanning tree. Prove that a graph is connected if and only if it has a spanning tree. **(8 Marks)**

7. (a) Define a group, Sub - group. Give one example each show that a group $(G*)$ is abelian iff $(a * b)^2 = a^2 * b^2$ **(8 Marks)**

(b) Define homomorphism and isomorphism between two groups. Give one example each. **(6 Marks)**

(c) Prove that an (m, n) encoding function $e : B^m \longrightarrow B^n$ can detect k or fewer errors if and only if its minimum distance is atleast $k + 1$. **(6 Marks)**

8. Write short notes on the following : **(5 Marks each)**

i) Normal forms

ii) Travelling Salesman Problem

iii) Group codes

iv) Recurrence relation

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MCA12

Page No. 1

NEW SCHEME

USN

First Semester M.C.A Degree Examination, July/August 2003

Master of Computer Applications
(New Scheme)

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Define inverse, converse and contrapositive of a conditional statement. Give one example each. **(6 Marks)**

(b) Find the possible truth value of p, q and r if

i) $p \rightarrow (q \vee r)$ *is FALSE*

ii) $p \wedge (q \rightarrow r)$ *is TRUE* **(6 Marks)**

(c) Obtain the principal conjunctive normal form of

i) $\sim (p \vee q)$ ii) $\sim (p \rightarrow q)$ **(8 Marks)**

2. (a) i) Show that for any two sets A and B $A - (A \cap B) = A - B$

ii) If $A \cap B = A \cap C$, must $B = C$? Explain your answer with example. **(6 Marks)**

(b) By mathematical induction prove that $(1+x)^n = 1 + nc_1 x + nc_2 x^2 + - - - - + x^n$ **(6 Marks)**

(c) A woman has 11 close relatives. In how many ways can she invite five of them to dinner

i) If two of the relatives are married and will not attend separately?

ii) If two of them are not on speaking terms and will not attend together? **(8 Marks)**

3. (a) Define symmetric, asymmetric and antisymmetric relations with one example each. **(6 Marks)**

(b) A relation R on a set A is called circular if aRb and bRc imply cRa. Show that R is reflexive and circular if and only if it is an equivalence relation. **(6 Marks)**

(c) Let $A = \{2, 3, 6, 12\}$ and Let R and S be the following relations on A

$$xRy \ iff \ \ 2/x - y \ and$$
$$xSy \ iff \ \ 3|x - y$$

Compute : i) $R \cap S$ ii) $\overline{R}$ iii) $\overline{S}^{1}$ iv) $R \cup S$ **(8 Marks)**

4. (a) Define transitive closure of a relation. If $R = \{(1,2),(2,3),(3,4),(2,1)\}$ is a relation on the set $A = \{1,2,3,4\}$ find the transitive closure of R using Warshall's algorithm. **(6 Marks)**

(b) Find an explicit formula for $a_n = 2a_{n-1} + 1$ *with* $a_1 = 7$ Use Backtracking technique. **(6 Marks)**

Contd.... 2

(c) Solve the recurrence relation $a_n = -6\,a_{n-1} - 9\,a_{n-2}$ given that $a_1 = 2.5,\ a_2 = 4.7$ **(8 Marks)**

5. (a) Let $A = B = C = R$, the set of all real numbers. Consider the following functions.
$f : A \to B$ *and* $g : B \to C$ defined by
$f(a) = 2a + 1$ *and* $g(b) = \frac{b}{3}$

i) Find $(fog)(-2)$ *and* $(gog)(-1)$

ii) Verity that $(gof)^{-1} = f^{-1}og^{-1}$ **(8 Marks)**

(b) Define even and odd permutations. Is the permutation

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$$ even or .odd

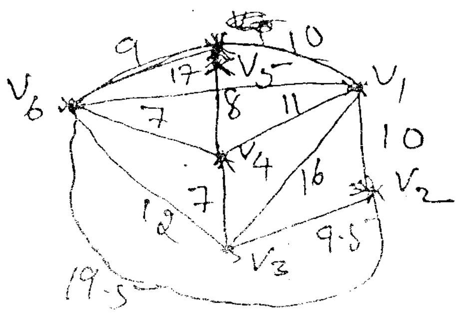
(6 Marks)

(c) Let $A = B = R$, the set of all real numbers. Let $f : A \to B$ *and* $g : B \to A$ given by $f(x) = 2x^3 - 1$ and $g(x) = \sqrt[3]{\frac{y}{2} + \frac{1}{2}}$. Show that f is a bijection from A to B and g is a bijection from B to A. **(6 Marks)**

6. (a) Define : i) Multigraph ii) Simple graph iii) Null graph
iv) Hamilterian graph. **(4 Marks)**

(b) Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degrees. **(8 Marks)**

(c) Using Prim's algorithm, find the shortest spanning tree in the below given graph. What is the weight of the shortest spanning tree? **(8 Marks)**



7. (a) Define a group. If $(G, *)$ is a group show that
i) $(a * b)^{-1} = b^{-1} * a^{-1}$ ii) G is abelian iff $(a * b)^2 = a^2 * b^2$ **(8 Marks)**

(b) If H and K are normal subgroups of a group G, show that $H \cap k$ is a normal subgroup of K. Is $H \cup k$ a normal subgroup of G? Explain your answer. **(6 Marks)**

(c) Define Monoid, subgroup and semigroup. Give examples. **(6 Marks)**

8. Explain the following with examples.

i) Pigeonhole principle.

ii) Predicates

iii) Hashing function.

iv) Minimum distance of code. **(4×5=20 Marks)**

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NEW SCHEME

MCA12

USN [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

First Semester M.C.A Degree Examination, January/February 2004

Master of Computer Applications

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.

1. (a) Let p, q and r denote propositions. Determine whether the following formulas are logically equivalent.

i) $p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (q \to r)$

ii) $p \to (q \vee r) \Leftrightarrow [> r \to (p \to > q)$ **(10 Marks)**

(b) If there was a ball game then travelling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time, therefore, there was no ball game. Show that these statements constitute a valid argument. **(6 Marks)**

(c) Explain quantifiers by means of an example. **(4 Marks)**

2. (a) Given $A = \{x|x$ is an integer and $1 \leq x \leq 5\}$, $B = \{3,4,5,17\}$ and $C = \{1,2,3....\}$, find $A \cap B$, $A \cap C$, $A \cup B$ *and* $A \cup C$. **(4 Marks)**

(b) Among 100 students, 32 study mathematics, 20 study physics and 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology, and 30 do not study any of the three subjects.

i) Find the number of students studying all three subjects.

ii) Find the number of students studying exactly one of the three subjects. **(6 Marks)**

(c) In a class of 100 students, 40 were boys. In how many ways a 10 - person committee be formed if

i) there must be an equal number of boys and girls in the committee.

ii) the committee consists of six boys and four girls or four boys and six girls. **(6 Marks)**

(d) Explain rules of sum, and rules of product. **(4 Marks)**

3. (a) Let R be a binary relation on the set of integers such that aRb if and only if a–b is an odd integer. What are the properties of R(Discuss at least six)? **(7 Marks)**

(b) Let R and S be the equivalence relations on a set A. Check whether $R \cap S$ and $R \cup S$ are equivalence relations? **(7 Marks)**

(c) Let $R = \{(1,2), (3,4), (2,2)\}$ and $S = \{(4,2), (2,5), (3,1), (1,3)\}$. compute $R^0 S$, $S^0 R$, $(R^0 S)^0 R$, $R^o R^o R$, $S^o S^o S$, R^4. **(6 Marks)**

4. (a) Solve the recurrence relation $a_r = a_{r-1} + a_{r-2}, a_0 = 1$. **(6 Marks)**

(b) Solve the recurrence relation $a_r = 3a_{r-1} + 2,\ r \geq 1$ by using generating functions. **(8 Marks)**

Contd.... 2

(c) Let $A = \{a, b, c, d\}$ *and* $B = \{1, 2, 3\}$ Determine whether the relation R from A to B is a function. If so, give its range.

i) $R = \{(a,1),\ (b,3),\ (c,1),\ (d,2)\}$

ii) $R = \{(a,1),\ (b,2),\ (c,1),\ (b,3),\ (d,2)\}$

iii) $R = \{(a,1),\ (b,1),\ (c,1),\ (d,1)\}$ **(6 Marks)**

5. (a) List all possible functions from $X = \{a, b, c\}$ *to* $Y = \{0, 1\}$ and indicate in each case whether the function is one to one, is on to, and is one to one on to. **(8 Marks)**

(b) Define i) an inverse function and ii) a characteristic function on a net. Give one example each. **(6 Marks)**

(c) Show that if seven numbers from 1 to 12 are chosen, then two of them will add up to 13. **(6 Marks)**

6. (a) Define an Eulerian graph and a Hamilton graph. Give one example each. **(6 Marks)**

(b) Show that in a tree, there exists exactly one path between every pair of vertices. **(4 Marks)**

(c) Explain prefix codes by means of an example. **(5 Marks)**

(d) Show that a connected graph has at least one spanning tree. **(5 Marks)**

7. (a) Define i) Semi group ; ii) Monoid and iii) a group. Explain how these algebraic systems differ from one another. **(6 Marks)**

(b) If $(G, \cdot)$ is an abelian group, show that

i) $(a \cdot b)^2 = a^2 \cdot b^2$ ii) $(a \cdot b)^n = a^n \cdot b^n,\ n \in Z^+$ **(7 Marks)**

(c) Let $G_1, \cdot, e_1)$ and $(G_2,\ *,\ e_2)$ be two groups. Let $f : G_1 \longrightarrow G_2$ be a homomorphism. Show that

i) $f(e_1) = e_2$ and ii) $f(a^{-1}) = [f(a)]^{-1}$ **(7 Marks)**

8. Write short notes on the following :

a) Tautology and contradiction

b) Group codes

c) Hashing functions

d) Konigsberg's bridge problem. **(5×4=20 Marks)**

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NEW SCHEME

MCA12

USN

First Semester M.C.A Degree Examination, July/August 2004

Master of Computer Applications

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: ***1. Answer any FIVE full questions.***
2. All questions carry equal marks.

1. a) Define the following logical operators.

i) Conjunction ii) Disjunction iii) Biconditional. **(4 Marks)**

(b) Show that $(q \to (p \vee c)) \Longleftrightarrow (q \wedge \neg p) \to c$. **(6 Marks)**

(c) Define tautology. Examine whether the following statement is a tautology :

$[(p \to q) \wedge \neg q] \to \neg p$. **(4 Marks)**

(d) Show that $p \to q$ and $q \to r$ imply $p \to r$. **(6 Marks)**

2. a) Express the following statements symbolically.
If there is a ball game, then travelling was difficult.
If they arrived on time, then travelling was not difficult.
They arrived on time.
Therefore, there was noball game.

Also show that the above statements constitute a valid argument. **(6 Marks)**

(b) If $u = \{1,2,3,4,...,10\}$, $A = \{1,2,3,5,6\}$, $B = \{3,5,6,7\}$ and $C = \{5,6,7,8\}$, Compute

i) $(A - B) \cap C$

ii) $\overline{A \cup B}$

iii) $(A \cap B) \oplus C$. **(6 Marks)**

(c) Prove the following :

i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(8 Marks)**

3. a) A survey of 260 college students revealed the following data :

64 had taken mathematics course,
94 had taken computer science course,
58 had taken business management course,
28 had taken mathematics and business management courses,
26 had taken mathematics and computer science courses,
22 had taken computer science and business management courses, and
14 had taken all the 3 courses.

Find the number of students who

i) had taken none of the three courses

ii) had taken only the computer science course. **(8 Marks)**

Contd.... 2

(b) Prove by induction :

$1.2 + 2.3 + ------ + n(n+1) = \frac{n(n+1)(n+2)}{3}$. **(4 Marks)**

(c) An urn contains 15 balls, 8 of which are red and 7 are black. In how many ways can 5 balls be selected so that

i) all 5 are red?

ii) all 5 are black ?

iii) 2 are red and 3 are black? **(4 Marks)**

(d) Show that if seven numbers are chosen from 1 to 12, then two of them will add up to 13. **(4 Marks)**

4. a) Let $A = \{1,2,3\}$. Let R and S be the relations on A whose matrices are

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

List the elements of

i) R ii) $\overline{R}$ iii) R^{-1} iv) $R \cap S$. **(4 Marks)**

(b) Let $A = \{a,b,c\}$ and $R = \{(a,a),(a,b),(b,a),(b,c),(c,a)\}$

Draw the digraph of R and obtain R^2. **(4 Marks)**

(c) Define an equivalence relation R on the set A.

If $A = \{1,2,3,4\}$ and $\{\{1,2\},\{3,4\}\}$ is a partition of A, obtain the equivalence relation R on A induced by the given partitions. **(5 Marks)**

(d) A relation R on a given set has the matrix :

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using Warshall's algorithm obtain the matrix of the relation which is the transitive closure of R. **(7 Marks)**

5. a) Using backtracking method, solve difference equation

$b_n = 2b_{n-1} + 1$ with the initial condition $b_1 = 7$. **(5 Marks)**

(b) Solve the homogeneous equation :

$a_n = 3a_{n-1} - 2a_{n-2}$ with $a_1 = 5$ and $a_2 = 3$. **(5 Marks)**

(c) Define the following with an example for each :

i) One-to-one function

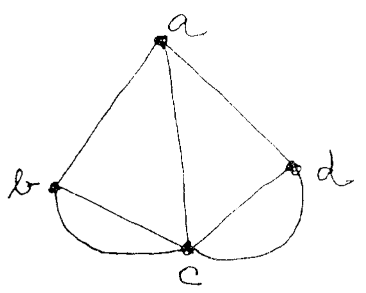
ii) Onto function

iii) Characteristic function of a set. **(6 Marks)**

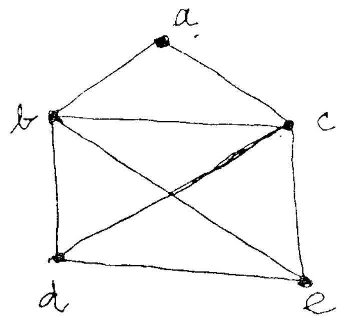
(d) If $P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$ Compute $(P_2 o P_2)^{-1}$. **(4 Marks)**

6. a) If $f : A \to B$ and $g : B \to C$ are one-to-one functions, prove that $g\ of$ is one-to-one. **(4 Marks)**

(b) State Euler's theorem for an undirected graph. Use the theorem to examine the existence or nonexistence of an Eulerian path in each of the following 2 graphs : **(8 Marks)**



Graph 1



Graph 2

(c) Define a tree. Give the relation $R = \{(a,b),(c,e),(f,a),(f,c),(f,d)\}$ defined on the set $A = \{a,b,c,d,e,f\}$, draw the digraph of R and hence show that R is a tree. **(4 Marks)**

(d) Define :

i) A Hamiltonian path in a graph.

ii) A cut-set in a graph. **(4 Marks)**

7. a) Define the following algebraic structures, giving an example for each :

i) Monoid ii) Semigroup iii) Group. **(6 Marks)**

(b) Let $(A, *)$ be a group with the identity e. Then, prove that

i) e is unique

ii) $(a * b)^{-1} = b^{-1} * a^{-1}$ for $a, b \in A$. **(7 Marks)**

(c) If $(A, *)$ is an abelian group, then prove that for all $a, b \in A$, $(a * b)^n = a^n * b^n$. **(7 Marks)**

8. a) State and prove Lagrange's theorem. **(8 Marks)**

(b) Let $(G, *)$ and $(G', *')$ be 2 groups with the identities e and e'. If $f : G \to G'$ is an isomorphism from G to G', prove that $e' = f(e)$. **(6 Marks)**

(c) Show that the $(2,5)$ encoding function $e : B^2 \to B^5$ defined by

$e(00) = 00000$

$e(01) = 01110$

$e(10) = 10101$

$e(11) = 11011$

is a group code. **(6 Marks)**

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NEW SCHEME

MCA12

USN

First Semester M.C.A Degree Examination, January/February 2005

Master of Computer Applications

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: ***1. Answer any FIVE full questions.***
2. All questions carry equal marks.

1. (a) Define tautology. Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(6 Marks)**

(b) Prove that, for any three propositions p, q, r

$$[(p \vee q) \to r] \longleftrightarrow [(p \to r) \wedge (q \to r)]$$ **(6 Marks)**

(c) Test whether the following argument is valid :

If I drive to work, then I will arrive tired
I am not tired when I arrive at work
There fore, I do not drive to work. **(4 Marks)**

(d) Write down the following proposition in symbolic form and find its negation:

All integers are rational numbers and some rational numbers are not integers. **(4 Marks)**

2. (a) Let $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,2,3,7\}$, $B = \{4,5,6,7\}$ $C = \{1,3,6\}$

Compute : i) $A \cap B$ ii) $A - B$ iii) $A \cap (B \cup C)$ iv) $\overline{A} \cup \overline{C}$ **(6 Marks)**

(b) Prove that, for any three sets A, B, C

i) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

ii) $A \cup (B \cup C) = (A \cup B) \cup C.$ **(8 Marks)**

(c) A computer company must hire 25 programmers to handle systems programming jobs and 40 programmers for applications programming. Of those hired, ten will be expected to perform jobs of both types. How many programmers must be hired. **(6 Marks)**

3. (a) Prove by induction

$$1^2 + 2^2 + 3^2 + \dots\dots\dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$ **(5 Marks)**

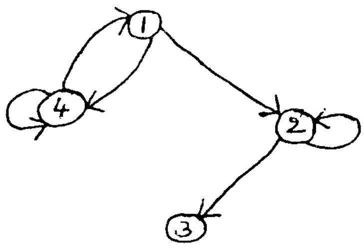
(b) Find the number of different permutations of the letters of the word MISSISSIPPI. **(4 Marks)**

Contd.... 2

(c) At a certain college, the housing office has decided to appoint, for each floor, one male and one female residential advisor. How many different pairs of advisors can be selected for a seven-story building from 12 male and 15 female candidates. **(6 Marks)**

(d) State the pigeonhole principle. Show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9. **(5 Marks)**

4. (a) Find the relation represented by the digraph given below. Also, write down its matrix. **(4 Marks)**



(b) If R and S are equivalence relations on a set A, then show that $R \cap S$ is an equivalence relation. **(6 Marks)**

(c) Let $R = \{(1,2)(3,4)(2,2)\}$ and $S = \{(4,2)(2,5)(3,1)(1,3)\}$ be relations on the set $A = \{1,2,3,4,5\}$. Find **(4 Marks)**

i) RoS ii) SoR iii) So (RoS) iv) Ro(SoR).

(d) Using Warshall's algorithm, find the transitive closure of the relation $R\{(1,2)(2,3)(3,3)\}$ on the set $A = \{1,2,3\}$. **(6 Marks)**

5. (a) Using back tracking method, solve the recurrence relation : **(5 Marks)**

$$a_n = a_{n-1} + 3 \text{ with the initial condition } a_1 = 2.$$

(b) Solve the homogeneous equation **(5 Marks)**

$$a_n = 5a_{n-1} - 6a_{n-2} \text{ with } a_1 = 5, a_2 = 13.$$

(c) Define : i) one-to-one function **(5 Marks)**

ii) characteristic function of a set. Give one example each.

(d) Let $A = \{x \mid x \text{ is real and } x \geq -1\}$ and $B = \{x \mid x \text{ is real and } x \geq 0\}$. Consider the function $f : A \to B$ defined by $f(a) = \sqrt{a+1}$, for all $a \in A$. Show that f is invertible and determine f^{-1}. **(5 Marks)**

6. (a) Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A = \{1,2,3,4,5,6\}$

i) Compute p^2 and p^3 **(4 Marks)**

ii) Find the smallest integer k such that $p^k = I_A$.

(b) Define an Euler graph and a Hamiltonian graph. Give one example for each. **(6 Marks)**

Contd.... 3

(c) Write the Prim's algorithm to find a minimal spanning tree of a weighted graph. **(5 Marks)**

(d) Define prefix code. Which of the following sets represent the prefix code? State reasons

$A = \{000, 001, 01, 10, 11\}$

$B = \{1, 00, 01, 000, 0001\}$ **(5 Marks)**

7. (a) Define a group. Show that in a group $(G, *)$

i) the identity element is unique

ii) every element of G has an unique inverse. **(8 Marks)**

(b) Prove that a group $(G, \cdot)$ is abelian $iff (a \cdot b)^2 = a^2 \cdot b^2$. **(6 Marks)**

(c) In a group $(G, \cdot)$ having more than one element, if $x^2 = x$ for every $x \in G$, prove that $G, \cdot)$ is abelian. **(6 Marks)**

8. (a) State and prove Lagrange's thoerm. **(8 Marks)**

(b) If f is a homomorphism from a group $(G_1, \cdot)$ to a group $(G_2, \cdot)$, then prove that $f[a^{-1}] = [f(a)]^{-1}$ **(6 Marks)**

(c) Show that the (2, 6) encoding frunion $e : B^2 \rightarrow B^6$ defined by

$e(00) = 000000, e(10) = 101010$
$e(01) = 010101, e(11) = 111111$ is a group code. **(6 Marks)**

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MCA12

USN

NEW SCHEME

First Semester M.C.A Degree Examination, July 2007

Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Let p and q be primitive statements for which the implication $p \to q$ is false. Determine the truth values of the following :
i) $p \wedge q$ ii) $\sim p \vee q$ iii) $q \to p$ iv) $\sim q \to \sim p$ **(05 Marks)**

b. Define tautology. Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (b \to r)$ is a tautology. **(05 Marks)**

c. Prove that, for any three propositions p, q, r
$[p \to (q \wedge r) \Leftrightarrow [(p \to q) \wedge (p \to r)]]$ **(05 Marks)**

d. Test whether the following argument is valid:
I will become famous or I will not become a musician.
I will become a musician.
Therefore, I will become famous. **(05 Marks)**

2 a. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 4, 6, 8}, B = {2, 4, 5, 9}, C = {x/x is a positive integer and $x^2 \le 16$} and D = {7, 8}. Compute
i) $A \cap C$ ii) $A \cup D$ iii) $\overline{D}$ iv) $B \oplus C$ **(06 Marks)**

b. Prove that, for any three sets A, B, C
i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(07 Marks)**

c. A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games, and 50 do not watch any of the three kinds of games.
i) How many people in the survey watch all three kinds of games?
ii) How many people watch exactly one of the sports? **(07 Marks)**

3 a. Prove by induction :
$1^2 + 3^2 + 5^2 + \cdots\cdots\cdots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ **(05 Marks)**

b. Find the number of different permutations of the letters of the word MASSASAUGA. In how many of these, all four A's are together? **(05 Marks)**

c. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations :
i) There is no restriction on the choice.
ii) Two particular persons will not attend separately. **(05 Marks)**

d. State the pigeonhole principle. Show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9. **(05 Marks)**

4 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if 'a' is a multiple of 'b'. Represent the relation R as a matrix and draw its digraph. **(05 Marks)**

4 b. If A = {1, 2, 3, 4} and R, S are relations on A defined by R = {(1, 2)(1, 3)(1, 4)}, S = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 3)(2, 4)} find $R \circ S$, $S \circ R$, R^2, S^2, $S \circ ($ (05 Ma

c. If R and S are equivalence relations on a Set A, then show that $R \cap S$ is an equivalence relation. **(05 Marks)**

d. Using Warshall's algorithm, find the transitive closure of the relation, R = {(1, 2)(2, 3)(3, 3)} on the set A = {1, 2, 3}. **(05 Marks)**

5 a. Using back tracking method, solve the recurrence relation : $a_n = a_{n-1} + 3$ with the initial condition $a_1 = 2$. **(05 Marks)**

b. Solve the homogeneous equation :
$a_n = 2a_{n-1} - a_{n-2}$ with $a_1 = 1.5$, $a_2 = 3$ **(05 Marks)**

c. Let A = {x / x is real and $x \geq -1$} and B = {x / x is real and $x \geq 0$}. Consider the function f : A → B defined by $f(a) = \sqrt{a+1}$, for all $a \in A$. Show that f is invertible and determine f^{-1}. **(05 Marks)**

d. Let A = B = C = R (set of real numbers), and f : A → B and g : B → C be defined by

$f(a) = 2a + 1$, $g(b) = \frac{1}{3}b$, $\forall a \in A, \forall b \in B$

Compute $g \circ f$ and show that $g \circ f$ is invertible. What is $(g \circ f)^{-1}$? **(05 Marks)**

6 a. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set A = {1, 2, 3, 4, 5, 6}.

i) Write p as a product of disjoint cycles.
ii) Compute p^{-1}.
iii) Compute p^2 and p^3. **(06 Marks)**

b. Define Euler graph and Hamiltonian graph. Give one example for each. Give an example for an Eulerian graph but not Hamiltonian. **(07 Marks)**

c. Define spanning tree of a graph. Find all the spanning trees of the graph given below. **(07 Marks)**

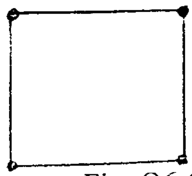

Fig. Q6 (c)

7 a. Define a group (G, *). Show that in a group (G, *)
i) The identity element is unique.
ii) Every element of G has an unique inverse. **(06 Marks)**

b. Define Abelian group. Prove that a group (G, •) is abelian iff $(a \cdot b)^2 = a^2 \cdot b^2$ **(07 Marks)**

c. Define subgroup. If H and K are subgroups of a group (G, •), prove that $H \cap K$ is a subgroup of G. Is $H \cup K$ a subgroup of G? Justify your answer. **(07 Marks)**

8 a. Define left and right cosets. State and prove Lagrange's theorem. **(08 Marks)**

b. Define group homomorphism. If 'f' is a homomorphism from a group $(G_1, \cdot)$ to a group $(G_2, \cdot)$, then prove that $f(a^{-1}) = [f(a)]^{-1}$. **(06 Marks)**

c. Define a group code. Show that the (2, 6) encoding function $e : B^2 \to B^6$ defined by,
e (00) = 000000, e(10) = 101010, e(01) = 010101, e(11) = 111111 **(06 Marks)**

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MCA12

USN

NEW SCHEME

First Semester MCA Degree Examination, Dec. 06 / Jan. 07

Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. State the rules for generating well-formed formula. Identify whether the following are well-formed formulae or not.

i) $(P \rightarrow (P \vee Q))$

ii) $\neg(P \cup Q)$

iii) $(P \rightarrow Q) \rightarrow (\wedge Q)$

iv) $((P \wedge Q) \rightarrow Q)$ **(06 Marks)**

b. Prove that for any three propositions P, Q, R

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ **(06 Marks)**

c. Obtain the principal disjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$ **(04 Marks)**

d. Obtain the principal conjunctive normal forms of S given by $(\neg P \rightarrow R) \wedge (Q \rightleftarrows P)$ **(04 Marks)**

2 a. Let A, B, C be any three sets. Then

i) Show that $(A - B) - C = A - (B \cup C)$

ii) Show that $(A - B) - C = (A - C) - (B - C)$ **(06 Marks)**

b. Show that $2^n \times 2^n - 1$ is divisible by 3 for all $n \geq 1$ by induction. **(07 Marks)**

c. A five person committee having members M_1, M_2, M_3, M_4 and M_5 is to select a President, Vice-President and secretary.

i) How many selections exclude M_1?

ii) How many selections include M_2 and M_3?

iii) How many selections are there in which M_5 is President? **(07 Marks)**

3 a. Let R be a binary relation from A to B. Define converse of R. If R is reflexive, is R^{-1} necessarily reflexive? If R is symmetric, is R^{-1} necessarily symmetric? If R is transitive, is R^{-1} necessarily transitive? **(07 Marks)**

b. Define a compatible relation. Let Z_+ be the set of all positive integers and R be a relation on Z_+ defined by $(a, b) \in R$ if and only if $a - b$ is divisible by S. Prove that R is compatible relation on Z_+ and also obtain corresponding cover of I_+. **(07 Marks)**

c. Define transitive closure. Write the Warshall's algorithm for finding the transitive closure. **(06 Marks)**

4 a. Define partitions of a set. Let R be a symmetric and transitive relation on a set A. Show that if for every $a \in A$, there exists $b \in A$, such that $(a, b) \in R$, then R is an equivalence relation. **(06 Marks)**

b. Solve the linear recurrence relation $a_r + a_{r-1} + a_{r-2} = 0$. **(06 Marks)**

c. Solve $a_r - 7a_{r-1} + 10a_{r-2} = 7(3^r)$, $r \geq 2$ **(08 Marks)**

Contd.... 2

5 a. Let $f : X \to Y$ be a function and $A \subseteq X$. Define restriction of f to A with one example. If $|X| > |Y|$ then prove that at least two elements of X has the same image in Y. **(07 Marks)**

b. Let f, g, h be functions from N to N, where N is the set of natural number so that $f(n) = (n+1)$, $g(n) = 2n$ and $h(n) = \begin{cases} 0 & n \text{ is even} \\ 1 & n \text{ is odd}, \quad n \in N \end{cases}$

Determine fof, foy, goh. **(06 Marks)**

c. Define a characteristic function of a set. Employing characteristic functions, prove that, for any three sets A, B, C.

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(07 Marks)**

6 a. Define a simple path and simple circuit of a graph. In a graph with n vertices, if there is a path from vertex V_1 to vertex V_2, then prove that there is a path of no more than $n-1$ edges from vertex V_1 to vertex V_2. **(07 Marks)**

b. Define an eulerian path. Prove that a graph possesses an eulerian path if it is connected and has either zero or two vertices of odd degree. **(07 Marks)**

c. Define a spanning tree. Write the algorithm to find minimum spanning tree. **(06 Marks)**

7 a. Define multigraph, simple graph, Hamilterian graph with one example each. **(06 Marks)**

b. Show that a tree with n vertices has $n-1$ edges. **(07 Marks)**

c. Define homomorphism of semigroups and monoids. Let G be a group with respect $*$. Prove that left and right cancellation law is true for all elements in G. **(07 Marks)**

8 a. Prove that any two right cosets are either identical or disjoint. **(06 Marks)**

b. If H and K are normal subgroups of a group G. Show that $H \cap K$ is normal subgroup of G. Is $H \cup K$ is normal subgroups of G? Explain your answer. **(06 Marks)**

c. Define kernel of homomorphism. Let f be a homomorphism of $(G,*)$ into $(\bar{G},\cdot)$ with kernel K. Then prove that K is a normal subgroup of G. **(08 Marks)**

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07MCA12

First Semester MCA Degree Examination, Dec. 07 / Jan. 08
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. State and prove the De Morgan's laws for union and intersection. **(06 Marks)**
b. Explain the proof templates with suitable examples. **(08 Marks)**
c. Derive the principle of inclusion – exclusion for three sets. **(06 Marks)**

2 a. State the principle of mathematical induction. Prove by mathematical induction that,
$$\sum_{i=0}^{n} a.r^i = \frac{a(1-r^{n+1})}{1-r}.$$ **(10 Marks)**
b. Define the Disjunctive and Conjunctive Normal forms. For the following formulas find equivalent formulas in CNF and DNF.
i) $(p \wedge q) \leftrightarrow (p \wedge r)$
ii) $((p \rightarrow q) \rightarrow r) \rightarrow p$ **(10 Marks)**

3 a. Define the terms predicates and quantification. Translate the following sentences into formulas using quantifiers:
i) Not all lawyers are judges.
ii) All lawyers admire some judges.
iii) Some lawyers are absent for the court.
iv) No lawyer earns more than the judge. **(10 Marks)**
b. Define tautology, contradiction and satisfiable sentences. Classify the following formulas into tautology, contradiction or satisfiable, using truth tables.
i) $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
ii) $((\neg p \vee q) \rightarrow (q \rightarrow p))$
iii) $\neg((p \wedge (p \rightarrow q)) \rightarrow q)$ **(10 Marks)**

4 a. Let R and S be binary relations on a set A then, prove that,
i) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$
ii) If $S \subseteq R$, then $S^{-1} \subseteq R^{-1}$ **(08 Marks)**
b. What is an equivalence relation? Test whether the relation R defined on set of integers I as follows:
$R = \{(x, y) \mid (x - y)$ is divisible by $5\}$ is an equivalence relation. **(06 Marks)**
c. Define the partial orderings and linear orderings and optimal elements in orderings with an example to each. **(06 Marks)**

5 a. Write the topological sorting algorithm for a partial order R on a set A with $|A| = n$. Apply the topological sorting algorithm to the partial order given below: **(10 Marks)**

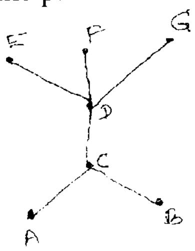

Fig. Q5 (a)

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5 b. Given a set A with $|A| = n$, and a relation R on A, let M denote the relation matrix for R. Then prove that

i) R is reflexive iff $I_n \leq M$.

ii) R is symmetric iff $M = M^{tr}$

iii) R is transitive iff $M^2 \leq M$. **(10 Marks)**

Support the proof with an example.

6 a. For each of the following functions $f : Z \to Z$, determine whether the function is one-to-one or onto. If the function is not onto, determine the range *f*(*Z*).

i) $f(x) = 2x - 3$

ii) $f(x) = x^2$ **(08 Marks)**

iii) $f(x) = x^2 + x$

b. State the pigeonhole principle. Show that if 8 people are in a room, at least two of them are born on the same day. **(06 Marks)**

c. Let $f : A \to B$ and $g : B \to C$. Prove that

i) If *f*, *g* are one-to-one, then $g \circ f$ is one-to-one. **(06 Marks)**

ii) If *f*, *g* are onto, then $g \circ f$ is onto.

7 a. Define the following with an example to each:

i) Bipartite graph.

ii) Subgraph

iii) Isomorphic graphs. **(10 Marks)**

iv) Eulerian circuit

b. Represent the following graph with adjacency list and adjacency matrix and find a Hamiltonian cycle: **(06 Marks)**

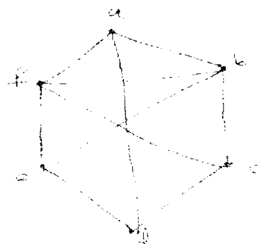

Fig. Q7 (b)

c. Define the following:

i) Konigsberg bridge problem. **(04 Marks)**

ii) Graph coloring.

8 a. Give a formal definition of a group. Prove that for every group G,

i) The identify of G is unique. **(08 Marks)**

ii) The inverse of each element of G is unique.

b. Define the following :

i) Group homomorphism.

ii) Abêlian group. **(06 Marks)**

iii) Subgroup.

c. Define the binary operation $\circ$ on Z by $x \circ y = x + y + 1$. Verify that (Z, $\circ$) is an abelian group. **(06 Marks)**

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07MCA12

First Semester MCA Degree Examination, June / July 08

Discrete Mathematics

Time: 3 hrs. Max. Marks: 100

Note : *Answer any FIVE full questions.*

1 a. Define power set with example. If a finite set A has n elements then prove that power set of A has 2^n elements. **(08 Marks)**

b. 30 cars were assembled in a factory. The options available were a radio, air conditioner, white wall tyres. It is known that 15 of the cars have radios, 8 of them have air conditioner and 6 of them have white wall tyres, 3 of them have all three options. Determine at least how many cars do not have any options at all. **(06 Marks)**

c. Define principle of mathematical induction and prove by mathematical induction that,

$$a + ar + ar^2 + \ldots\ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r},\ r \neq 0.$$ **(06 Marks)**

2 a. Define Tautology and contradiction. Prove that for any propositions p, q, r the compound proposition

$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a Tautology. **(06 Marks)**

b. Prove that the following logical equivalences with out using truth tables:

i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$ **(05 Marks)**

c. Find the disjunctive and conjuctive normal form of the following proposition:

$p \wedge (p \to q)$. **(04 Marks)**

d. Prove that the following argument is valid:

$\forall x, [p(x) \to \{q(x) \wedge r(x)\}]$

$\forall x, [p(x) \wedge s(x)]$

$\therefore \forall x, [r(x) \wedge s(x)]$ **(05 Marks)**

3 a. Define partition of a set with example. If $A = Z$ (set of all integers) and $R = \{(a,b) \in A \times A / 2 \text{divides} (a-b)\}$ is a relation on A, then show that R is an equivalence relation. **(08 Marks)**

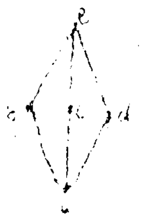
b. Define matrix representation of relation with example. If R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)} is a relation on A = {1, 2, 3, 4} then find digraph of R and list in-degree, out-degree of all vertices. **(08 Marks)**

c. If R is a relation from A to B, S is a relation from B to C then show $(S \bullet R)^{-1} = R^{-1} \bullet S^{-1}$ **(04 Marks)**

4 a. Define poset with example and if $(A, \leq)$ and $(B, \leq)$ be posets then show that $(AXB, \leq)$ is a poset with partial order '$\leq$' defined by $(a,b) \leq (a',b')$ If $a \leq a'$ in A and $b \leq b'$ in B. **(08 Marks)**

b. Define Hasse diagram of a poset with example and show that if A is a finite non empty poset with partial order $\leq$ then A has at least one max element and at least one min element. **(08 Marks)**

c. Define isomorphic Lattices with example. Which of the following Hasse diagrams are Lattices? **(04 Marks)**



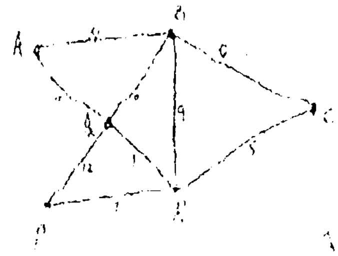


Fig. Q4 (c)-(i) Fig. Q4 (c) (ii) Fig. Q7 (c)

5 a. Define identity function and constant function with example and let A=B=Z (set of all integers) and $f : A \rightarrow B$ be a function defined by $f(a) = a+1$, $\forall a \in A$ find f is one to one or onto or both or neither (08 Marks)

b. Let $f : A \rightarrow B$ be a function, then prove the following:

i) f^{-1} is a function from B to A iff f is one to one

ii) If f^{-1} is a function then the function f^{-1} is also one to one

iii) f^{-1} is every where defined iff f is onto.

iv) f^{-1} is onto iff f is everywhere defined (08 Marks)

c. If n pigeons are assigned to m pigeonholes then prove that one of the pigeonhole must contain at least $\left\lfloor \frac{n-1}{m} + 1 \right\rfloor$ pigeons. (04 Marks)

6 a. Define regular graph and bipartite graphs with examples. Show that in a complete graph of n vertices the degree of every vertex is (n-1) and the total number of edges is $\frac{n(n-1)}{2}$. (08 Marks)

b. Discuss Konigsberg bridge problem (06 Marks)

c. For a graph G with n vertices and m edges, if δ is the minimum, Δ is the maximum of the degree of vertices then show that $\delta \le \frac{2m}{n} \le \Delta$. (06 Marks)

7 a. Show that the complete graph of five vertices is non-planar. (06 Marks)

b. Show that a tree with n vertices has (n-1) edges. (06 Marks)

c. Write Kruskal's algorithm for minimal spanning tree. Find a minimal spanning tree for a weighted graph below by prims algorithm. (08 Marks)

8 a. If * be a binary operation on a set A and * satisfies following properties for any a, b, c ∈ A

i) $a = a*a$ ii) $a*b = b*a$ iii) $a*(b*c) = (a*b)*c$.

Define a relation ≤ on A by a ≤ b iff a = a*b then show that (A, ≤) is a poset and for all a,b ∈ A. Then GLB(a,b) = a*b. (06 Marks)

b. Define homomorphic, isomorphism and monoids with example and let (S,*) and (T,*) be monoids with respective identities e & e', let f : S → T, be an isomorphism then show that f(e) = e'. (08 Marks)

c. Define homomorphism with example. If f is a homomorphism from a commutative semigroup (S, *) onto a semigroup (T, *), then show that (T, *) is also a commutative. (06 Marks)

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USN

07MCA12

First Semester MCA Degree Examination, Dec.08/Jan.09
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. Define i) Power set; ii) Symmetric difference of two sets with examples. **(06 Marks)**

b. Prove that for any three sets A, B, C

i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **(07 Marks)**

c. In a survey of 254 college students, the following data were obtained: 64 had taken Mathematics course, 94 had taken Physics course, 58 had taken chemistry course, 28 had taken both Mathematics and chemistry course, 26 had taken both Mathematics and Physics course, 22 had taken both Physics and chemistry course, and 14 had taken all three types of courses.

Find how many of these students had taken none of the three courses? **(07 Marks)**

2 a. If n is a positive integer, prove that $1.2 + 2.3 + 3.4 + \ldots\ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ using mathematical induction. **(06 Marks)**

b. Define tautology and contradiction. Construct a truth table to show that $(p \leftrightarrow q) \leftrightarrow ((p \to q) \wedge (q \to p))$ is a tautology. **(07 Marks)**

c. i) Find the DNF of the formula $(\neg(p \to q)) \to (q \wedge \neg r)$, using truth table.

ii) Find the CNF of $p \leftrightarrow q$ using truth table. **(07 Marks)**

3 a. Define universal and existential quantifiers negate the following statement.

$\forall x \exists y\, [(p(x,y) \wedge q(x,y)) \to r(x,y)]$ **(06 Marks)**

b. For a fixed integer n>1, prove that the relation "Congruent modulo n" is an equivalence relation on the set of all integers, Z. **(07 Marks)**

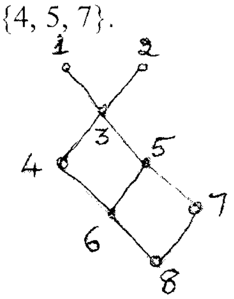
c. Let A = {1, 2, 3, 4, 5, 6, 12}. On A, define the relation R by aRb if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(07 Marks)**

4 a. Define Isomorphic ordered sets and well ordered sets. Give an example for each set. **(06 Marks)**

b. What is Lattice? With an example verify the laws of Lattice. **(07 Marks)**

c. What are i) Upper bound and lower bound; ii) Supremum and Infimum.

Let S = {1, 2, 3, ...8} be ordered as shown in the Hasse diagram below. Find the above i) and ; ii) for the set A = {4, 5, 7}. **(07 Marks)**



5 a. Define inverse function and Composition of functions. Illustrate with examples. **(06 Marks)**

b. State the generalized Pigeonhole Principle. What is the minimum number of students needed in a discrete mathematics class to be given that at least six will receive the same grade, if there are five possible grades A, B, C, D and E? **(07 Marks)**

c. Prove that Evens = $\{n \in N : n = 2k \text{ for some } K \in N\}$ is countably infinite and also define countably infinite set. **(07 Marks)**

6 a. Define: Sub graph, spanning sub graph and Induced sub graph of a graph g. Give examples. **(06 Marks)**

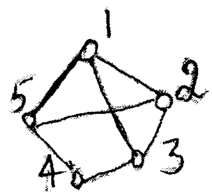
b. P.T in any graph, the number of odd vertices is even. **(07 Marks)**

c. Prove that a graph T is a tree if and only if any two vertices of T are joined by a unique path **(07 Marks)**

7 a. Define i) Rooted tree, ii) Binary tree and iii) Binary search tree with examples. **(06 Marks)**

b. What is graph coloring and chromatic number of a graph G. Give an illustration, Show that a tree is 2 chromatic. **(07 Marks)**

c. For the following graph write the adjacency matrix and adjacency list. Also find a , Hamiltonian cycle for the graph. **(07 Marks)**

G:


8 a. Define group and an abelian group. Show that, the set of all fourth roots of unity that is $\{1, -1, i, -i\}$ is a group under usual multiplication. **(06 Marks)**

b. Prove that H is a subgroup of G if and only if, for all a, $b \in H$, $ab^{-1} \in H$. **(07 Marks)**

c. Define : i) Normal subgroup; ii) homomorphism, with example; iii) Isomorphism of groups. **(07 Marks)**

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07MCA12

First Semester MCA Degree Examination, June-July 2009
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Define power set of a set. If A = {1, 2, {{1, 2}}} determine P(A), the power set of A. How many elements does P(P(A)) have? **(06 Marks)**

b. How many natural numbers between 1 and 30000000 (including 1 and 30000000) are divisible by 2, 3 or 5? (Use principles of inclusion and exclusion). How many are not divisible by 2, 3 or 5? **(07 Marks)**

c. Prove by induction for $n \geq 0$, 3 divides n^3+2n **(07 Marks)**

2 a. Define a tautology and contradiction. Show that $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology and $(p \rightarrow (q \vee r)) \wedge ((\sim q) \wedge (\sim r) \wedge p)$ is a contradiction. **(08 Marks)**

b. Define conjuctive normal form and disjuctive normal form. Write the conjuctive and disjuctive normal forms for,
$\sim (p \rightarrow q) \rightarrow (q \wedge \sim r)$ **(08 Marks)**

c. Let U be the set of all problems on a comprehensive list of problems in science. Define
P(x) : x is a mathematics problem
Q(x) : x is difficult, R(x) : x is easy and
S(x) : x is unsolvable. Translate into English sentences each of the following:
i) $\forall x P(x)$ ii) $\forall x (S(x) \rightarrow P(x))$ iii) $\exists x (S(x) \wedge \sim P(x))$ iv) $\exists x (\sim Q(x) \wedge \sim R(x))$ **(04 Marks)**

3 a. If A = {1, 2, 3, 4, 5}, give an example of a relation R on A that is,
i) Reflexive and symmetric but not transitive.
ii) Reflexive and transitive but not symmetric.
iii) Symmetric and transitive but not reflexive. **(06 Marks)**

b. For A = {1, 2, 3, 4}, let R and S be the relations on A defined by R = {(1, 2), (1, 3), (2, 4), (4, 4)} and S = {(1, 1), (1, 2), (1, 3), (2, 3), (2, 4)}. Find ROS, ROR, ROROR, SOSOS, $\overline{S \cup R}$, $R^{-1} \cap S^{-1}$ and SOR. **(07 Marks)**

c. Define R on A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} by xRy if x-y is a multiple of 5. Show that R is an equivalence relation. Determine the partition of A introduced by R. **(07 Marks)**

4 a. Define a partially ordered set. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

b. If A is a finite poset, prove that A has
i) at least one maximal element.
ii) at least one minimal element.
iii) at most one maximal element.
iv) at most one minimal element. **(08 Marks)**

c. If $(L, \leq)$ is a lattice, prove that
i) $a \vee b = b$ iff $a \leq b$
ii) $a \wedge b = a$ iff $a \leq b$
ii) $a \wedge b = a$ iff $a \vee b = b$ **(06 Marks)**

5 a. Define injective function, bijective function and surjective function. Give one example each. **(06 Marks)**

b. Let f : R→ R be a function defined as f(x) = 2x+8 and G : R→R defined as $g(y) = \frac{y-8}{2}$.
Prove that both f and g are identity functions on R. **(07 Marks)**

c. State and prove generalized Pigeon Hole principle. Use the theorem to find the number of students in a class of strength 89 to have a birthday in the same month. **(07 Marks)**

6 a. Define a complete graph, a regular graph, a bipartite graph and induced subgraph Hamiltonian graph and Euler graph. **(06 Marks)**

b. In any graph, show that the number of odd vertices is even. **(07 Marks)**

c. Explain Konigsberg bridge problem. **(07 Marks)**

7 a. Let G be a connected graph. Show that G has an Eulerian circuit if and only if each vertex is even. **(06 Marks)**

b. Construct the following graphs:

i) A graph G such that $\overline{G}$ is not connected.
ii) A 4-regular graph of six vertices.
iii) A tree with five vertices.
iv) A complete graph with 6 vertices. **(08 Marks)**

c. For every tree T = (V, E), if $|V| \geq 2$ then show that T has at least two pendant vertices. **(06 Marks)**

8 a. For every graph (G. *) prove that

i) The inverse of each element of G is unique.
ii) $(ab)^{-1} = b^{-1}a^{-1}$, for all $a, b \in G$. **(08 Marks)**

b. Show that any group G is abelian iff $(ab)^2 = a^2b^2$ for $a, b \in G$. **(06 Marks)**

c. Let (G, *) and $(G', *')$ be groups with identities e and e' respectively. If $f : G \rightarrow G'$ is a homomorphism prove that,

i) $f(e) = e'$
ii) $f(a^{-1}) = (f(a))^{-1}$ for all $a \in G$. **(06 Marks)**

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07MCA12

First Semester MCA Degree Examination, Dec.09/Jan.10

Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Prove that for any 3 sets A B C

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(06 Marks)**

b. How many natural numbers between 1 and 30,000,000 (including 1 and 30,000,000) are divisible by 2, 3, or 5? **(08 Marks)**

c. Prove or disprove by using suitable proof template

i) For any 3 sets A B C, $A \cup B = A \cup C$ then $B = C$

ii) $3n + 2$ is odd then n is odd. **(06 Marks)**

2 a. Prove by mathematical induction

$P(n) : n^3 - n$ is divisible by 3 where $n \in Z^+$. **(06 Marks)**

b. Obtain disjunctive normal form for the formula

$P \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$. **(06 Marks)**

c. Let p, q and r denote propositions. Determine whether following are logically equivalent

$P \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$ **(08 Marks)**

3 a. If R and S are binary relations on a set X, prove that

$(R)^{-1} = R$

$(R \cup S)^{-1} = R^{-1} \cup S^{-1}$. **(06 Marks)**

b. If R_1 and R_2 are equivalence relations on a set x then prove that $R_1 \cap R_2$ is an equivalence relation. Give a counter example to show that $R_1 \cup R_2$ need not be an equivalence relation. **(08 Marks)**

c. Let x = {-3, -2, -1, 0, 1, 2, 3}. For any x, y ∈ x define $x R_y$ if $x^2 < y^2$ or $x = y$. Show that R is a partial ordering on x. Draw the Hasse diagram for R. **(06 Marks)**

4 a. Find reflexive, symmetric and transitive closure for the relation R on x = {a, b, c} defined as R = {(a, b) (b, c)}. **(06 Marks)**

b. i) Define a lattice. Show that set of all divisors of 70 form a lattice.

ii) Let S = {a, b, c, d, e} be ordered as in Fig. Q4(b)(ii). Find all possible consistent enumerations $f : S \rightarrow \{1, 2, 3, 4, 5\}$. **(08 Marks)**

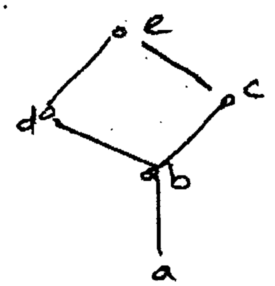

Fig. Q4(b)(ii)

c. If (A, R) is a poset and A is finite, show that A has both maximal and minimal element. **(06 Marks)**

5 a. Which of the following functions are bijections?
i) $f : |R \to |R$ given by $f(x) = 2x - 3$
ii) $g : |R \to |R$ given by $g(x) = x^2 - 2$. **(06 Marks)**

b. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if f and g are bijections then g o f is also a bijection. f and g are defined over R (set of reals) $f(x) = x^2 + 3x - 8$ $g(x) = \cos^2 x - 1$ compute (f o g) (x) and (g o f)(x). **(08 Marks)**

c. Prove that set of prime positive integers is countably infinite. **(06 Marks)**

6 a. State extended (generalised) pigeonhole principle. Prove that in a group of 44 people atleast four must be born in the same month. **(06 Marks)**

b. Let A = B = C = |R and consider the functions $f : A \to B$ and $g : B \to C$ defined by $f(a) = 2z + 1$; $g(b) = b/3$, verify $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(08 Marks)**

c. Define Hashing function. Find the memory locations assigned by the Hashing function h(k) = k(mod 101) to the record of insurance company customers with social security numbers i) 104578690 ii) 432222187. **(06 Marks)**

7 a. Prove that a finite connected graph G is Eulerian if the only if each vertex has even degree. **(06 Marks)**

b. Define planar graphs. Show that k_5 is nonplanar graph with least number of vertices and $k_{3,3}$ is nonplanar graph with least number of edges. **(08 Marks)**

c. Discuss the following :
i) Four color theorem
ii) Traveling salesman problem. **(06 Marks)**

8 a. Define Monoid, group and semigroup, with an example each. **(06 Marks)**

b. If (G, $*$) is a group, prove that identity and inverse elements of group are unique. **(08 Marks)**

c. Define normal subgroup. Prove that every subgroup of an abelian group is normal subgroup. **(06 Marks)**

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First Semester MCA Degree Examination, May/June 2010
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Prove that for any 3 sets A, B and C
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(08 Marks)**

b. If A and B are any 2 sets, prove that $A - B = A - (A \cap B)$ **(04 Marks)**

c. Among 100 students, 32 study literature, 20 study psychology, 45 study economics, 15 study literature and economics, 7 study literature and psychology, 10 study psychology and economics, 30 do not study any of the three subjects, find
i) The number of students studying all the 3 subjects
ii) Number of students studying exactly one of the three subjects. **(08 Marks)**

2 a. Prove that for any three prepositions P, Q and R
$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ **(06 Marks)**

b. Obtain the principle conjunctive normal form of
$(\sim P \rightarrow R) \wedge (Q \leftrightarrow P)$ **(06 Marks)**

c. If P and Q are propositions for which $P \rightarrow Q$ is false, determine the truth values of
$P \wedge Q$, $\sim P \vee Q$, $Q \rightarrow P$, $\sim Q \rightarrow \sim P$. **(08 Marks)**

3 a. Let A = { 1, 2, 3, 4, 6 } and R be a relation on A defined by aRb iff a is a multiple of b. Represent the relation R as a matrix and draw its digraph. **(06 Marks)**

b. A = { 1, 2, 3, 4} R, S be relation on A
R = {(1, 2), (1, 3), (2, 4) (4, 4)}
S = {(1, 1), (1, 2) (1, 3), (1, 4), (2, 3), (2, 4)}
Find RoS, SoR, R^2, S^2, So(RoS). **(06 Marks)**

c. Using Warshall's algorithm find the transitive closure of the relation,
R = {(1, 2), (2, 3), (3, 4), (2, 1)} on A = { 1, 2, 3, 4 } **(08 Marks)**

4 a. Define a partial ordering, with an example. **(04 Marks)**

b. Let L be a lattice. Then for a, b ∈ L
show that $a \vee b = b$ iff $a \subseteq b$
$a \wedge b = a$ iff $a \subseteq b$ **(08 Marks)**

c. Consider the partial order of divisibility on the set A. Draw the Hasse diagram of the poset A = { 1, 2, 3, 5, 6, 10, 15, 30 } **(08 Marks)**

5 a. Define a function, bijection, invertible function and characteristic function. **(08 Marks)**

b. Let A = B = C = R
$f : A \rightarrow B$, $g : B \rightarrow C$ defined by $f(a) = 3a - 1$ and $g(b) = b^2 + 1$.
Find gof(-2) and fog(-2). **(06 Marks)**

c. Let A = { x | x is real; x ≥ - 1} ; B = { x | x is real; x ≥ 0}.
Define $f : A \rightarrow B$ by $f(a) = \sqrt{a+1}$, a ∈ A. Show that f is invertible and find f^{-1}. **(06 Marks)**

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6 a. Discuss Konigsberg bridge problem. **(06 Marks)**

b. Prove that a given connected graph is an Eulerian graph iff all vertices of h are of even degree. **(08 Marks)**

c. Show that the number of odd vertices in a graph is even. **(06 Marks)**

7 a. Show that a connected graph G is a tree iff there exists exactly one path between every pair of vertices in G. **(10 Marks)**

b. Define spanning tree. Find a minimum spanning tree for the graph as shown in Fig.Q7(b).

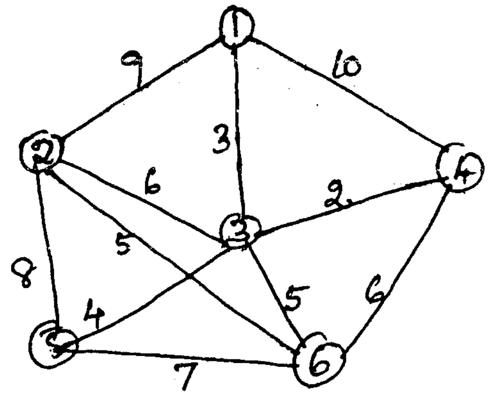

Fig.Q7(b) **(10 Marks)**

8 a. Prove that in a group, identity and inverse are unique. **(08 Marks)**

b. If (H, o), (K, o) are subgroup of (G, o), prove that (H∩K, o) is also a subgroup of G. **(06 Marks)**

c. If $a^2 = a$, $\forall$ a $\in$ (G, o), prove that (G, o) is an abelian group. **(06 Marks)**

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